NOTATION

ψ, stream function, m^2/sec; ν, coefficient of kinematic viscosity, m^2/sec; T, temperature, °C; $N = \nu_0/\nu_w$; μ, coefficient of dynamic viscosity; Pr, Prandtl number; f, dimensionless stream function; η, self-similar variable; $Gr_x = g\beta(T_w - T_0)x^3/\nu_0^2$, Grasshof number; ρ, density of the fluid, kg/m^3; α, heat transfer coefficient, W/(m^2·K); δ_t, thickness of thermal boundary layer; F_{fr}, force of friction. Subscripts: 0, temperature of the fluid at some distance from the wall; w, temperature of the wall; *, controlling temperature; max, maximum longitudinal velocity.

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DETERMINING THE MAXIMUM VELOCITY OF A GAS BEING DISCHARGED OUT OF A NOZZLE

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It is proposed to calculate the maximum discharge velocity of a chemically neutral real gas out of a Laval nozzle with consideration given to the latent heat of vapor formation of the substance in the expansion of the gas within the nozzle to total condensation, as well as to calculate the parameters which correspond to the saturation line. An equation has been derived to determine the maximum velocity of the discharge gas from an energy point of view.

The velocity at which a gas is discharged out of the diffuser of a Laval nozzle as the gas expands to a pressure of $p_n = 0$ and a temperature $T_n = 0$ and, consequently, given a gas enthalpy of $i_n = 0$ this discharge velocity is usually referred to as the maximum velocity [1]. The value of this maximum velocity is determined from the equation of conservation for the energy of the gas as the latter expands within a nozzle, without phase transition:

$$W_1^2 = 2i_c. \tag{1}$$

With T_c = const and a variable p_c for an ideal gas i_c = const, and, therefore, in actual practice, when T_c = const, the value of W_1 is assumed to be independent of p_c [2].

Let us examine the question as to whether the velocity defined by Eq. (1) is the only maximum velocity for a chemically neutral real gas. A basis for the validity of the formulation of such a problem can be found, as demonstrated in [3, 4], that the heat of vapor formation evolved in the condensation of the metal vapors generates (within the diffuser of the nozzle) an increase in the velocity of the uncondensed gas.

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The results from [3, 4] allow us to write the equation of velocity (or the law of energy conservation) in the following form for the flow of a chemically neutral substance within whose process of gas-phase expansion we find the formation of a liquid phase:

$$W^2 = 2(i_c - i_n + \psi r - e). \tag{2}$$

As the gas expands from p_c, T_c to $0 \leq p_n < p_c$, $0 \leq T_n < T_c$, the value of i_n changes within the limits $0 \leq i_n < i_c$, and given corresponding regime expansion parameters the substance may either remain gaseous ($\psi = 0$) or change partially into the liquid phase ($0 < \psi < 1$), or change completely into the liquid phase ($\psi = 1$). The material in the nozzle diffuser can be held to be gaseous in the strict sense of the word, provided that there exists no thermodynamic conditions for condensation, which is equivalent to the expansion of the gas in the nozzle to the parameters $p_n \neq p_s$, $T_n \neq T_s$. As $p_n \to p_s$, $T_n \to T_s$ (and, consequently, $i_n \to i_s$) we find from (2) the equation for the velocity of the gas on expansion to parameters close to the conditions of condensation (the saturation line):

$$W_2^2 = 2(i_c - i_s). \tag{3}$$

The expansion of the gas in the diffuser of thenozzle to $p_n = p_s$, $T_n = T_s$ creates thermodynamic conditions of condensation. Regardless ofthe value of ψ, we will assume that the gas changes into the liquid phase is discontinuous fashion during the expansion to the parameters $p_n = p_s$, $T_n = T_n$, and the initial velocity of liquid-phase motion is equal to the velocity of the gas determined from Eq. (3), and in the subsequent combined motion of the two phases their velocities are equal. This assumption is validated by the fact that, first of all, the usually observed supersaturation of the gas in nozzles and jets corresponds to exceedingly small droplet dimensions of 10^{-8}-10^{-9} m [5] and, secondly, it has been demonstrated in [3] that the velocity of droplets having a dimension of $\sim 2 \cdot 10^{-6}$ m differs from the velocity of the gas by no more than 1%.

The expenditures of energy in the gas phase of the flow on increasing the initial kinetic energy

$$E_0 = 0{,}5\,\psi m_g W_2^2 \tag{4}$$

the condensate mass $m = \psi m_g$ to the final kinetic energy

$$E_{out} = 0{,}5 m_g W^2 \tag{5}$$

and the same condensate mass at the nozzle outlet all amount to

$$e = (E_{out} - E_0)\, m_g^{-1} = 0{,}5\psi (W^2 - W_2^2). \tag{6}$$

Having substituted (5), (6), and (3) into (2), after simple transformations we will derive the equation for the velocity of the two-phase (gas + liquid) flow of substance at the nozzle outlet, with consideration given to the consumption of gas-phase energy on maintaining the velocity of the condensate at a level equal to the velocity of the gas phase:

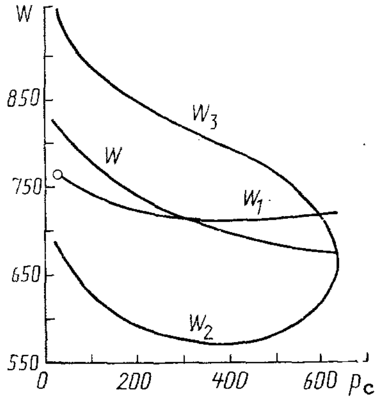

Fig. 1. Velocities W, W_1, W_2, and W_3 [m·sec^{-1}] of air discharged out of a nozzle as a function of p_c [10^5 Pa] when T_c = 300 K.

$$W^2 = 2\,[i_c - i_s + \psi r\,(1 + \psi)^{-1}]. \tag{7}$$

For all of the substances the parameters p_s and T_s of a saturation line limited by a melting point having the coordinates p_m and T_m and a critical point with p_{cr} and T_{cr}, correspond strictly with one another [7] and are attained in the process of gas expansion within the nozzle when p_c and T_c combine ahead of the nozzle, as defined by the equation of isentropy:

$$p_s/p_c = (T_s/T_c)^{\gamma(\gamma-1)^{-1}}. \tag{8}$$

Consequently, the region in which the parameters of gas expansion within a nozzle exist, and for which the values of W and W_2 can be determined, is bounded from below by the line of saturation, and it is bounded on the right and on the left by the lines of isentropic expansion of the gas, these lines ascending from the boundary points of the saturation line, these points having the coordinates p_m, T_m and p_{cr}, T_{cr}. At the critical point $r = 0$, so that $W_2 = W_{cr}$ is in the expansion of the gas to the critical point. At the melting point $r = r_{max}$ of the substance it may consequently be assumed that as the gas expands in the nozzle diffuser to the melting point, at which $\psi = 1$, $i_s = i_m$, the velocity of substance flow reaches the following energy-dependent maximum value:

$$W^2_{max} = 2\,(i_c - i_m + 0{,}5 r_{max}). \tag{9}$$

We can see from (9) that with total condensation of the gas in the nozzle diffuser ($\psi = 1$) half of the evolved heat of vapor formation is spent to accelerate the condensate to the velocity of the gas.

As an example, the figure shows the results obtained (with utilization of the data from [7], i.e., with consideration given to the real properties of the gas) from the calculation of W_1 and W_2 as a function of p_c when $T_c = 300$ K for air ($\gamma = 1.4$), which is assumed to be the optimum working fluid of a gas-reactive system [2]. Here we also find the results from a calculation of W for the case in which $\psi = 1$ on the basis of Eq. (7) and for W_3 when $e = 0$ and $\psi = 1$, on the basis of Eq. (2), i.e., in the absence of expenditures of energy on acceleration of the condensate. The calculations were carried out under the condition that the air, in accordance with (8), expands from the pressure $p_c = (15.9\text{-}637.4)\cdot 10^5$ Pa within the chamber to the parameters of the saturation line at the nozzle outlet, limited by the fusion point for air ($p_m = 0.07\cdot 10^5$ Pa, $T_m = 64$ K) and the critical point ($p_{cr} = 37.7\cdot 10^5$ Pa, $T_{cr} = 132.5$ K).

We can see from the figure that when we take into consideration the real properties the values of W, W_1, and W_3 for the air increase with a reduction in p_c.

The figure identifies the point for the maximum value of $W_{1\,max} = 773$ m/sec. We can see that W and W_3 can assume values greater than $W_{1\,max}$ and, in particular, $W_{max} = 830$ m/sec, $W_{3\,max} = 947$ m/sec. Consequently, the concept of the maximum gas discharge velocity of a nozzle can have various meanings, since its magnitude is determined by the initial and final parameters of expansion and the level of condensation which accompanies the expansion of the gas in the nozzle. We can employ the energy-related maximum velocity defined by Eq. (9) as a concept to characterize the energy possibilities of a chemically neutral real gas. The theoretically limiting energy potentials of a chemically neutral real gas serve to characterize the value of $W_{3\,max}$.

NOTATION

T_c and T_n, absolute temperature of the gas within the chamber and in the nozzle; p_c and p_n, gas pressure in the chamber and in the nozzle; i_c and i_n, gas enthalpy in the chamber and in the nozzle; ψ, mass condensate fraction; r, latent heat of vapor formation; e, expenditures of gas-phase energy in the flow to maintain the velocity of the condensate at a level equal to the velocity of the material flow within the diffuser ofthe nozzle; T_s, p_s, and i_s, parameters of dry steam at the saturation line; T_m, p_m, and i_m, parameters of the substance at the melting point; T_{cr} and p_{cr}, parameters of the substance at the critical point; γ, isentropy exponent.

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